Fuzzy Quantifiers: A Natural Language Technique for Data Fusion

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Abstract – *Fuzzy quantifiers like 'almost all' and 'about half' abound in natural language. They are used for describing uncertain facts, quantitative relations and processes. An implementation of these quantifiers can provide expressive and easy-to-use operators for aggregation and data fusion, but also for steering the fusion process on a higher level through a safe transfer of expert-knowledge expressed in natural language. However, existing approaches to fuzzy quantification are linguistically inconsistent in many common and relevant situations. To overcome their deficiencies, we developed a new framework for fuzzy quantification, DFS. We first present the axioms of the theory, intended to formalize the notion of 'linguistic adequacy'. We then argue that the models of the theory are plausible from a linguistic perspective. We present three computational models and discuss some of their properties. Finally we provide an application example based on image data.*

Keywords: Fuzzy quantifiers, linguistic data fusion, weighted aggregation.

1 Introduction

There are many statistical approaches to data fusion available, some of them have been applied to real-world tasks with great success (e.g. [1, 2]). They suffer, however, from two main drawbacks: (a) statistical models are normally hard to establish and (b) human knowledge is hard to incorporate into the models and/or the fusion process. Natural language (NL) holds the potential to express this kind of human knowledge. NL would be an ideal candidate for modeling and steering the fusion process. Firstly, knowledge about the data sources (sensors etc.) can be expressed in NL statements; e.g. by describing the conditional reliability of a sensor through **if-then** rules. The fusion criterion itself can also be expressed in NL. However, an interpretation of NL statements requires appropriate semantic devices. NL concepts (expressed by nouns, verbs, adjectives) often lack clear boundaries. A practical model of such non-idealized concepts is provided by linguistic terms of fuzzy logic. In addition, NL quantifiers like **almost all** or **many** must be dealt with, which are frequently used to express approximate aggregation. Here we will focus on fuzzy quantifiers, because these are the tools of fuzzy logic that model linguistic fusion operators.

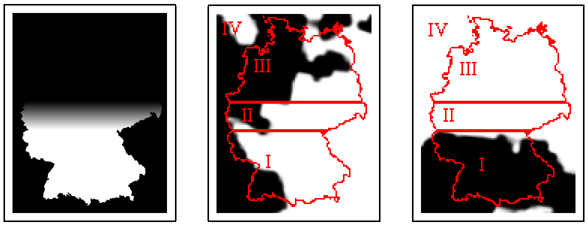
Conjunction k_1 **and** ... **and** k_m corresponds to: **all** criteria $k_1, \ldots, k_m$ are satisfied
Disjunction k_1 **or** ... **or** k_m corresponds to: **at least one** criterion $k_1, \ldots, k_m$ is satisfied
Fuzzy quantification $\widetilde{Q}(W, G)$ corresponds to: $\widetilde{Q}$ **important** criteria are **satisfied** $\widetilde{Q} \in \{$**almost all**, **many**, **about ten**, ...$\}$: fuzzy quantifier W: degree of importance, G: degree of validity

Table 1: Fuzzy quantifiers and weighted aggregation

Let us recall some concepts of fuzzy set theory. A *fuzzy subset* X of a base set $E \neq \varnothing$ assigns to each $e \in E$ a membership grade $\mu_X(e) \in [0, 1]$. An n*-ary fuzzy quantifier* $\widetilde{Q}$ on E assigns an interpretation $\widetilde{Q}(X_1, \ldots, X_n) \in [0, 1]$ to all fuzzy subsets $X_1, \ldots, X_n$ of E. Table 1 illustrates key properties that make fuzzy quantifiers suited for information fusion. Assuming a set E of criteria, and fuzzy subsets W of importance and G of their fulfillment grades, each two-place quantifier $\widetilde{Q}$ defines a weighted aggregation criterion $\widetilde{Q}$ *important criteria are satisfied*, expressed as $\widetilde{Q}(W, G)$, where $\widetilde{Q}$ is, for example, **almost all** etc.

This approach provides a natural account of weighted aggregation. It relies on linguistic fusion operators which are familiar to all users of information systems, and which can be applied for technical fusion purposes in the same way as in everyday language.

Apparently, the success of this approach is highly dependent on the model of quantification used, which must be linguistically plausible for the fusion results to convey the intended semantics. However, an evaluation of approaches to fuzzy quantification for their linguistic adequacy, has produced negative results in all cases [3]. The known models – i.e. the Σ-Count approach [4], FG-Count approach [4], OWA approach [5], and FE-Count approach [6] – are either too weak (cannot model the interesting cases) and/or



(a) (b) (c)

Figure 1: Results of OWA approach for criterion *At least 60 percent of Southern Germany are cloudy.* (a) Fuzzy region 'Southern Germany', relevant pixels white; (b) desired result: 1, OWA: 0.1; (c) desired result: 0, OWA: 0.6. The results show an undesirable dependency on cloudiness grades in regions III and IV, which do not belong to Southern Germany at all.

implausible from linguistic considerations. For an example in which one of the approaches fails, consider Fig. 1, which depicts a result of OWA with importance qualification [5]. More counter-examples, covering all approaches, can be found in [3]. It appears that non-monotonic quantifiers (*about half of the criteria are satisfied*) are notoriously difficult, as well as the important case of multi-place quantification, e.g. to handle the weights in *almost all important criteria are satisfied*.

2 The linguistic theory of fuzzy quantification

In order to profit from the knowledge of linguists, we decided to discuss fuzzy quantifiers in the framework of the *linguistic* theory of quantification. The Theory of Generalized Quantifiers [7] (TGQ) rests on a simple but expressive model of two-valued quantifiers, which provides a uniform representation for absolute and proportional quantifiers, unrestricted and restricted quantification (i.e. involving importances), and even for multi-place quantifiers like *more* $A's$ *than* $B's$ *are* $C's$, composite quantifiers like *most* A*'s and* B*'s are* C*'s or* D*'s*, and non-quantitative examples like *John* or *almost all married* X*'s are* Y*'s*. Nonetheless, TGQ was not developed with fuzzy sets in mind. Hence all quantifiers and argument sets involved (e.g. weights in importance qualification) must be crisp. TGQ is therefore not (directly) suited for real-world applications like linguistic fusion, which need to handle imperfect data and non-idealized NL concepts. In order to incorporate the notion of fuzziness, we identify a number of cornerstones of a principled theory:

1. *Improved Representation through* n*-ary semi-fuzzy quantifiers*
   Fuzzy quantifiers are often hard to define because the familiar concept of cardinality of crisp sets is not applicable to their fuzzy arguments. We propose a simplified concept which embeds all quantifiers of TGQ: An n-ary *semi-fuzzy quantifier* on E assigns an interpretation $Q(Y_1, \ldots, Y_n) \in [0, 1]$ to all *crisp* subsets of E. Because semi-fuzzy quantifiers accept crisp input only, they are easier to define than fuzzy quantifiers. The usual crisp cardinality is applicable to the arguments and can be used to define the interpretation of semi-fuzzy quantifiers.

2. A *quantifier fuzzification mechanism* (QFM) $\mathcal{F}$ assigns to each semi-fuzzy quantifier Q a fuzzy quantifier $\mathcal{F}(Q)$ with the same arity and base set; it hence extends the quantifier to fuzzy arguments. An adequate QFM should preserve all properties of linguistic relevance and should comply with important constructions on (semi-)fuzzy quantifiers. We enforce this by stating a set of axioms for 'admissable' QFMs, the *DFS axioms*.

3. We should find *models of the axioms*, i.e. 'reasonable' choices of $\mathcal{F}$;

4. *Efficient algorithms* must be developed for implementing the resulting operators.

Following these requirements, we have developed an axiomatic theory of fuzzy quantification known as 'DFS theory'. We first present the 6 axioms that are required:

1. **Correct generalisation**, i.e. $\mathcal{F}(Q)(X_1, \ldots, X_n) = Q(X_1, \ldots, X_n)$ whenever $X_1, \ldots, X_n$ are crisp (the condition can be restricted to $n \le 1$).
   Rationale: a semi-fuzzy quantifier Q has crisp arguments only, while $\mathcal{F}(Q)$ accepts fuzzy sets as well. If all arguments are crisp, Q and $\mathcal{F}(Q)$ must match.

2. **Membership assessment.** The two-valued quantifier defined by $\pi_e(Y) = 1$ if $e \in Y$ and $\pi_e(Y) = 0$ otherwise for crisp Y, has the obvious fuzzy counterpart $\widetilde{\pi}_e(X) = \mu_X(e)$ for fuzzy subsets of E. We require that $\mathcal{F}(\pi_e) = \widetilde{\pi}_e$.
   Rationale: Membership assessment can be modelled through quantifiers. For each $e \in E$, we define a two-valued quantifier π_e which checks if e is present in its argument. Similarly, $\widetilde{\pi}_e$ returns to which grade e is contained in its argument. It is natural that π_e be mapped to $\widetilde{\pi}_e$, which plays the same role in the fuzzy case.

3. **Dualisation.** We require that $\mathcal{F}$ preserves dualisation of quantifiers, i.e.

$$\mathcal{F}(Q')(X_1, \ldots, X_n) = \widetilde{\neg}\, \mathcal{F}(Q)(X_1, \ldots, X_{n-1}, \widetilde{\neg}\, X_n)$$

   holds for all fuzzy arguments X_i, provided that $Q'(Y_1, \ldots, Y_n) = \widetilde{\neg}\, Q(Y_1, \ldots, Y_{n-1}, \neg Y_n)$ for all crisp Y_i.
   Rationale: Obviously, a phrase like *all* X*'s are* Y*'s*

should have the same result as *it is not the case that some X's are not Y's*.

4. **Union.** We require that $\mathcal{F}$ preserves unions of arguments, i.e.

$$\mathcal{F}(Q')(X_1, \ldots, X_{n+1}) \\ = \mathcal{F}(Q)(X_1, \ldots, X_{n-1}, X_n \mathbin{\widetilde{\cup}} X_{n+1})$$

whenever $Q'(Y_1, \ldots, Y_{n+1}) = Q(Y_1, \ldots, Y_{n-1}, Y_n \cup Y_{n+1})$.
Rationale: It should not matter whether *many X's are Y's or Z's* is computed by evaluating $\mathcal{F}(\textbf{many})(X, Y \mathbin{\widetilde{\cup}} Z)$ or by computing $\mathcal{F}(Q)(X, Y, Z)$ with $Q(X, Y, Z) = \textbf{many}(X, Y \cup Z)$.

5. **Monotonicity in arguments.** We require that $\mathcal{F}$ preserve monotonicity in arguments, i.e. if Q is nondecreasing/nonincreasing in the i-th argument, then $\mathcal{F}(Q)$ has the same property. When combined with the other axioms, the condition can be restricted to the case that Q is nonincreasing in its n-th argument.
Rationale: The statement *all men are tall* must express a stricter condition compared to *all young men are tall*.

6. **Functional application.** Finally we require that $\mathcal{F}$ be compatible with a construction called 'functional application', i.e.

$$\mathcal{F}(Q')(X_1, \ldots, X_n) = \mathcal{F}(Q)(f_1'(X_1), \ldots, f_n'(X_n))$$

whenever Q' is defined by $Q'(Y_1, \ldots, Y_n) = Q(f_1(Y_1), \ldots, f_n(Y_n))$, where $f_1', \ldots, f_n'$ are obtained from the induced extension principle of $\mathcal{F}$, see [8].
Rationale: The axiom ensures that $\mathcal{F}$ behaves consistently across domains.

A QFM $\mathcal{F}$ which satisfies these axioms is called a determiner fuzzification scheme (DFS). If $\mathcal{F}$ induces the standard negation $\neg x = 1 - x$ and extension principle $\mu_{\hat{\hat{f}}(X)}(e) = \sup\{\mu_X(e') : f(e') = e\}$, then it is a *standard DFS*. These constitute the natural class of standard models of fuzzy quantification. A large number of properties of linguistic or logical relevance are entailed by the above axioms: If $\mathcal{F}$ is a DFS, then

- *$\mathcal{F}$ induces a reasonable set of fuzzy propositional connectives*, i.e. $\widetilde{\neg}$ is a strong negation $\widetilde{\wedge}$ is a t-norm, $\widetilde{\vee}$ is an s-norm etc., see [8];

- *$\mathcal{F}$ is compatible with the negation of quantifiers*. Hence

$$\mathcal{F}(Q')(X_1, \ldots, X_n) = \widetilde{\neg}\mathcal{F}(Q)(X_1, \ldots, X_n)$$

if $Q'(Y_1, \ldots, Y_n) = \widetilde{\neg} Q(Y_1, \ldots, Y_n)$, and *some tall men are lucky* is equivalent to *it is not true that no tall man is lucky* in $\mathcal{F}$;

- *$\mathcal{F}$ is compatible with the formation of antonyms*. Hence

$$\mathcal{F}(Q')(X_1, \ldots, X_n) = \mathcal{F}(Q)(X_1, \ldots, X_{n-1}, \widetilde{\neg} X_n)$$

if $Q'(Y_1, \ldots, Y_n) = Q(Y_1, \ldots, Y_{n-1}, \neg Y_n)$.
E.g., *all tall men are bald* is equivalent to *no tall men is not bald* in $\mathcal{F}$;

- *$\mathcal{F}$ is compatible with intersections*. This means that

$$\mathcal{F}(Q')(X_1, \ldots, X_{n+1}) \\ = \mathcal{F}(Q)(X_1, \ldots, X_{n-1}, X_n \mathbin{\widetilde{\cap}} X_{n+1})$$

if Q' is defined by $Q'(Y_1, \ldots, Y_{n+1}) = Q(Y_1, \ldots, Y_{n-1}, Y_n \cap Y_{n+1})$. Hence the meanings of *at least two X's are Y's* and *the set of X's that are Y's contains at least two elements* coincide in $\mathcal{F}$;

- *$\mathcal{F}$ is compatible with argument permutations*. Hence

$$\mathcal{F}(Q')(X_1, \ldots, X_n) = \mathcal{F}(Q)(X_{\beta(1)}, \ldots, X_{\beta(n)})$$

whenever $Q'(Y_1, \ldots, Y_n) = Q(Y_{\beta(1)}, \ldots, Y_{\beta(n)})$, where β is a permutation of $\{1, \ldots, n\}$. In particular, $\mathcal{F}$ preserves symmetries, and *about* 50 *X's are Y's* is equivalent to *about* 50 *Y's are X's* in $\mathcal{F}$.

- Finally, *$\mathcal{F}$ is compatible with argument insertion*, i.e.

$$\mathcal{F}(Q')(X_1, \ldots, X_n) = \mathcal{F}(Q)(X_1, \ldots, X_n, A)$$

whenever $Q'(Y_1, \ldots, Y_n) = Q(Y_1, \ldots, Y_n, A)$, for a fixed crisp argument $A \in \mathcal{P}(E)$. For example, the meanings of *many (married X)'s are Y's* and *(many married) X's are Y's* coincide in every DFS.

In addition, every DFS maps quantitative (automorphism-invariant) quantifiers like **almost all** to quantitative fuzzy quantifiers; and non-quantitative quantifiers like **John** or **most married** to non-quantitative fuzzy quantifiers. Moreover, DFSes are *contextual*, i.e. the interpretation of $\mathcal{F}(Q)(X_1, \ldots, X_n)$ only depends on the behaviour of Q inside the ambiguity ranges $\mathrm{core}(X_i) \subseteq Y_i \subseteq \mathrm{support}(X_i)$, where $\mathrm{core}(X_i)$ denotes the elements with unity membership and $\mathrm{support}(X_i)$ those with non-zero membership. Every DFS is also known to *preserve extension*, i.e. insensitive to the domain as a whole: we expect the interpretation of *most tall people are bald* not to depend on the chosen domain, as long as it is large enough to contain the fuzzy subsets **tall** and **bald** of interest. For a comprehensive discussion of the adequacy issue, see [8].

3 Models for linguistic data fusion

We now present three models of the theory, which are suited to support linguistic data fusion. These models use the fuzzy median,

$$\text{med}_{\frac{1}{2}}(u_1,u_2) = \begin{cases} \min(u_1,u_2) & : \min(u_1,u_2) > \frac{1}{2} \\ \max(u_1,u_2) & : \max(u_1,u_2) < \frac{1}{2} \\ \frac{1}{2} & : \text{else} \end{cases}$$

for all $u_1, u_2 \in [0,1]$. $\text{med}_{\frac{1}{2}}$ can be extended to an operator which accepts arbitrary subsets of $[0,1]$, viz $\text{m}_{\frac{1}{2}} X = \text{med}_{\frac{1}{2}}(\inf X, \sup X)$ for all subsets of $[0,1]$. In the following, we need the cut range $\mathcal{T}_\gamma(X) \subseteq \mathcal{P}(E)$ of a fuzzy subset X at the cutting level $\gamma \in [0,1]$, which corresponds to a symmetrical, three-valued cut of X at γ:

$$\mathcal{T}_\gamma(X) = \{Y \subseteq E : X_\gamma^{\min} \subseteq Y \subseteq X_\gamma^{\max}\},$$

$$X_\gamma^{\min} = \begin{cases} X_{\geq \frac{1}{2}+\frac{1}{2}\gamma} & : \gamma \in (0,1] \\ X_{>\frac{1}{2}} & : \gamma = 0 \end{cases}$$

$$X_\gamma^{\max} = \begin{cases} X_{>\frac{1}{2}-\frac{1}{2}\gamma} & : \gamma \in (0,1] \\ X_{\geq\frac{1}{2}} & : \gamma = 0 \end{cases}$$

Here $X_{\geq\alpha} = \{e \in E : \mu_X(e) \geq \alpha\}$ denotes α-cut, and $X_{>\alpha} = \{e \in E : \mu_X(e) > \alpha\}$ the strict α-cut. (γ can be thought of as a parameter of 'cautiousness'.) In order to interpret fuzzy quantifiers for any fixed choice of the cutting parameter, we stipulate

$$Q_\gamma(X_1,\ldots,X_n) = \text{m}_{\frac{1}{2}}\{Q(Y_1,\ldots,Y_n) : Y_i \in \mathcal{T}_\gamma(X_i)\},$$

The results at the cautiousness levels must be aggregated. We hence define

$$\mathcal{M}(Q)(X_1, \ldots, X_n) = \int_0^1 Q_\gamma(X_1, \ldots, X_n)\, d\gamma\,.$$

It can be shown that $\mathcal{M}$ is a standard DFS. $\mathcal{M}$ is practical because it is continuous in arguments and quantifiers, i.e. robust against slight changes or noise in the arguments and the quantifier. However, the integral used in the definition of $\mathcal{M}$, is not the only possible way of abstracting from γ. The necessary and sufficient conditions on aggregation mappings $\mathcal{B}$ which make $\mathcal{M}_\mathcal{B}(Q)(X_1, \ldots, X_n) = \mathcal{B}((Q_\gamma(X_1, \ldots, X_n))_{\gamma\in[0,1]})$ a DFS are stated in [8]. This investigation also revealed that there exists a model $\mathcal{M}_{\text{CX}}$ with unique adequacy properties from a linguistic perspective. It is the *only standard model* which permits the compositional interpretation of adjectival restriction by a fuzzy adjective, like in *almost all young A's are B's*, and hence guarantees that $\mathcal{M}_{\text{CX}}$(**almost all young**)(A,B) = $\mathcal{M}_{\text{CX}}$(**almost all**)(**young** $\cap A, B)$. It also preserves the convexity of absolute quantitative quantifiers like **about 10**. In addition, it is continuous in arguments and quantifiers and hence robust against noise. It further propagates fuzziness, i.e. less specific input cannot result in more specific outputs (for additional properties see [8]). $\mathcal{M}_{\text{CX}}$ can be defined in terms of the cut ranges and median-based aggregation, but also in the following more compact form.

$$\begin{aligned}&\mathcal{M}_{\text{CX}}(Q)(X_1, \ldots, X_n)\\ &= \sup\{Q^L_{V,W}(X_1, \ldots, X_n) : V_1 \subseteq W_1, \ldots, V_n \subseteq W_n\}\end{aligned}$$

where

$$\begin{aligned}Q^L_{V,W}(X_1, \ldots, X_n) &= \min(\Xi_{V,W}(X_1, \ldots, X_n),\\ &\quad \inf\{Q(Y_1,\ldots,Y_n) : V_i \subseteq Y_i \subseteq W_i\}\\ \Xi_{V,W}(X_1, \ldots, X_n) &= \min_{i=1}^{n} \min(\inf\{\mu_{X_i}(e) : e \in V_i\},\\ &\quad \inf\{1-\mu_{X_i}(e) : e \notin W_i\})\,.\end{aligned}$$

$\mathcal{M}_{\text{CX}}$ consistently generalises the Sugeno integral/basic FG-count approach to arbitrary n-place quantifiers without any monotonicity requirements.

The class of known models has been broadened by abstracting from the median-based aggregation. Q_γ is then replaced with a pair of mappings which specify upper and lower bounds on the quantification results for all choices of the Y_i in the cut ranges:

$$\begin{aligned}\top_{Q,X_1,\ldots,X_n}(\gamma) &= \sup\{Q(Y_1, \ldots, Y_n) : Y_i \in \mathcal{T}_\gamma(X_i)\}\\ \bot_{Q,X_1,\ldots,X_n}(\gamma) &= \inf\{Q(Y_1, \ldots, Y_n) : Y_i \in \mathcal{T}_\gamma(X_i)\}\,.\end{aligned}$$

In [9], the DFSes definable by $\mathcal{F}_\xi(Q)(X_1, \ldots, X_n) = \xi(\top_{Q,X_1,\ldots,X_n}, \bot_{Q,X_1,\ldots,X_n})$ have been investigated and the corresponding conditions on ξ have been formalized. Our last example $\mathcal{F}_{\text{owa}}$ is representative of this new type of models.

$$\begin{aligned}\mathcal{F}_{\text{owa}}(Q)(X_1, \ldots, X_n) &= \tfrac{1}{2}\int_0^1 \top_{Q,X_1,\ldots,X_n}(\gamma)d\gamma\\ &\quad + \tfrac{1}{2}\int_0^1 \bot_{Q,X_1,\ldots,X_n}(\gamma)d\gamma\,.\end{aligned}$$

$\mathcal{F}_{\text{owa}}$ is a standard DFS. It extends the Choquet integral/basic OWA approach to multiplace and non-monotonic quantifiers. Its continuity ensures some stability against noise. $\mathcal{F}_{\text{owa}}$ does not propagate fuzziness and is hence inferior to $\mathcal{M}_{\text{CX}}$ from an adequacy perspective (unspecific input can produce more specific output). Nevertheless, $\mathcal{F}_{\text{owa}}$ can be advantageous if the inputs are overly fuzzy, because it still discerns cases in which models that propagate fuzziness ceise to be informative.

4 Conclusion

Fuzzy quantifiers are, in principle, a powerful method for combining information. These linguistic fusion operators are easy to use and to understand. In particular, it is much easier to incorporate expert knowledge into the fusion process by means of linguistic descriptions than it is, for example, to construct statistical models. Morevover, the linguistic approach holds the potential to specify 'fusion plans', i.e. *what* is to be fused *when* and *how* [10]. The existing approaches to fuzzy quantification, however, give rise to implausible results in relevant situations. The most useful tool for improvement we found to be the linguistic theory of quantification, TGQ. Our theory of fuzzy quantification, DFS, consistently extends TGQ to incorporate gradual

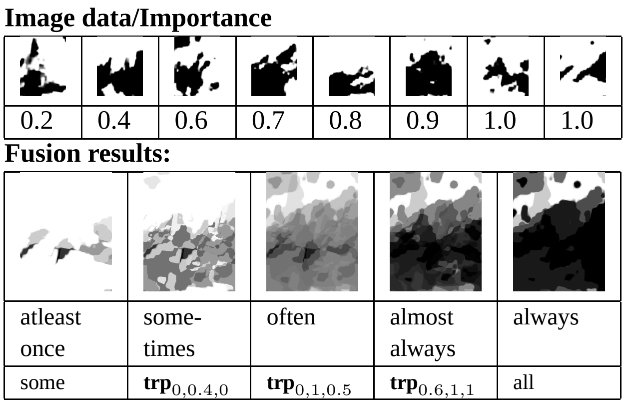


Figure 2: Image sequence and fusion results for various choices of the criterion *Q-times cloudy in the last days.* Regions that meet the criterion are depicted white.

interpretations and fuzzy arguments. Due to its axiomatic foundation, all models of the theory exhibit the desired adequacy properties. We have discovered models which consistently extend existing approaches to non-monotonic and multiplace quantification ($n > 1$); viz the basic FG-count approach/Sugeno integral and OWA approach/Choquet integral. These models are computational, and algorithms for the most common quantifiers have been developed [3]. We conclude with an example which demonstrates the suitability of the models to fuse image data, see Fig. 2. The data is comprised of a sequence of images, depicting fuzzy regions of cloudiness grades at given points of time. The images are qualified by importance to a fuzzy temporal condition, 'in the last days'. The DFS $\mathcal{M}$ was used and a quantifier **trp** (for trapezoidal) was applied that is given by

$$\mathbf{trp}_{a,b,c}(Y_1, Y_2) = \begin{cases} t_{a,b}(|Y_1 \cap Y_2|/|Y_1|) & : \quad Y_1 \neq \varnothing \\ c & : \quad Y_1 = \varnothing \end{cases}$$

$$t_{a,b}(z) = \begin{cases} 0 & : \quad z < a \\ \frac{z-a}{b-a} & : \quad a \leq z \leq b \\ 1 & : \quad z > b\,. \end{cases}$$

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