

Exercises for the Lecture

Techniques in Artificial Intelligence

18.1.2013 – Sheet 3- Solutions

1) Bayesian Networks I

Based on two simple weather parameters, clear or cloudy sky, and rising or falling barometer, perceived in the morning, the weather in the afternoon (dry or rainy) shall be predicted. For the prediction based on classical probability, a complete joint probability distribution is needed as given in the following table:

Morning		Afternoon	Probability
X1: sky	X2: barometer	X3: rain?	P(X1,X2,X3)
Clear	Rising	Dry	0.40
Clear	Rising	Rain	0.07
Clear	Falling	Dry	0.08
Clear	Falling	Rain	0.10
Cloudy	Rising	Dry	0.09
Cloudy	Rising	Rain	0.11
Cloudy	Falling	dry	0.03

a) How many events for the three variables exist for this joint probability distribution?

8.

Only 7 entries are given, one line is missing in the table, but the joint probability for “cloudy, falling, rain” can be calculated and equals to 0.12

b) Calculate $P(X3=dry \mid X1=clear, X2=rising)$

In the following, abbreviations like “clear” for $X1=clear$, etc. are used. This can be done, when there is no risk of misinterpretation.

$$P(dry \mid clear, rising) = \frac{P(dry,clear,rising)}{P(clear,rising)} = \frac{0.4}{0.47} = 0.85$$

c) Calculate $P(X3=rain | X1=cloudy)$

$$P(rain | cloudy) = \frac{P(rain,cloudy)}{P(cloudy)} = \frac{0.11+0.12}{0.09+0.11+0.03+0.12} = \frac{0.23}{0.35} = 0.66$$

d) What would you do, if the lowest row in the table is not given?

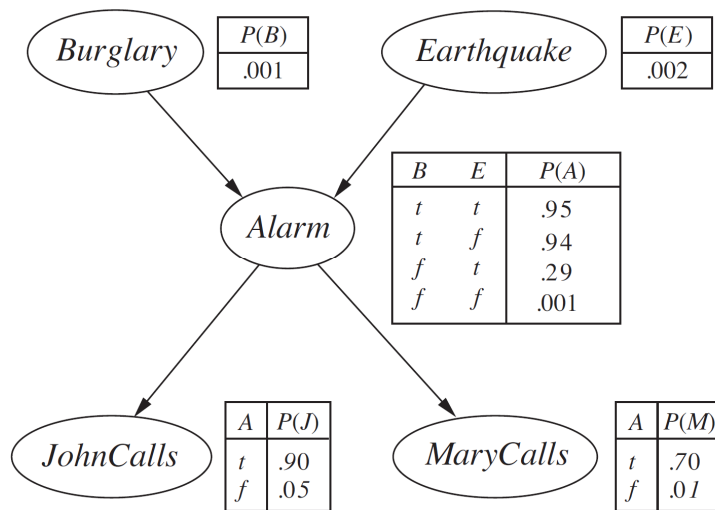
This means, that 2 entries are missing.

Without additional information, you need to assume an equal distribution.

As the missing probability is $0.03+0.12=0.15$, the assumption would be to set both entries to $0.15/2 = 0.075$. In case of more missing entries, the total missing “mass” needs to be divided by the number of missing entries.

2) Bayesian Networks II

Consider the burglary/earthquake example from the lecture



a) Calculate the missing unconditioned probabilities $P(A)$, $P(J)$, $P(M)$

$$\begin{aligned}
 P(A) &= P(A, B, E) + P(A, \neg B, E) + P(A, B, \neg E) + P(A, \neg B, \neg E) \\
 &= P(A | B, E)P(B, E) + P(A | \neg B, E)P(\neg B, E) + P(A | B, \neg E)P(B, \neg E) \\
 &\quad + P(A | \neg B, \neg E)P(\neg B, \neg E) \\
 &= 0.95 * 0.001 * 0.002 + 0.29 * 0.999 * 0.002 + 0.94 * 0.001 * 0.998 + 0.001 * 0.999 \\
 &\quad * 0.998 \\
 &= 0.025
 \end{aligned}$$

$$P(J) = P(J, A) + P(J, \neg A) = P(J|A) * P(A) + P(J|\neg A) * P(\neg A) \\ = 0.9 * 0.0025 + 0.005 * (1 - 0.0025) = 0.052$$

$$P(M) = P(M, A) + P(M, \neg A) = P(M|A) * P(A) + P(M|\neg A) * P(\neg A) \\ = 0.7 * 0.0025 + 0.01 * (1 - 0.0025) = 0.0117$$

b) Calculate P(J|A)

0.9

c) Calculate P(J|B)

$$P(J|B) = \frac{P(J, B)}{P(B)} = P(J|A) \frac{P(A|B)P(B)}{P(B)} + P(J|\neg A) \frac{P(\neg A|B)P(B)}{P(B)} \\ = P(J|A)P(A|B) + P(J|\neg A)P(\neg A|B)$$

$$P(A|B) = \frac{P(A, B)}{P(B)}, \quad P(\neg A|B) = \frac{P(\neg A, B)}{P(B)}$$

$$P(A, B) = P(A, B, E) + P(A, B, \neg E) = P(A|B, E)P(B, E) + P(A|B, \neg E)P(B, \neg E)$$

$$P(A|B) = \frac{P(A|B, E)P(B, E) + P(A|B, \neg E)P(B, \neg E)}{P(B)} = P(A|B, E)P(E) + P(A|B, \neg E)P(\neg E)$$

The latter equation uses the independence of P(E) and P(B).

$$P(\neg A|B) = P(\neg A|B, E)P(E) + P(\neg A|B, \neg E)P(\neg E)$$

And finally,

$$P(J|B) = P(J|A)((P(A|B, E)P(E) + P(A|B, \neg E)P(\neg E)) \\ + P(J|\neg A)((P(\neg A|B, E)P(E) + P(\neg A|B, \neg E)P(\neg E)) \\ = 0.9 * (0.95 * 0.002 + 0.94 * 0.998) + 0.05 * (0.05 * 0.002 + 0.06 * 0.998) = 0.849017$$

Be careful! The red 0.05=P(J|¬A) is directly read from the CPT of the node A, the other probabilities like P(¬A| something) can be calculated by 1-P(A| something)

d) Calculate P(B|J)

$$P(B|J) = \frac{P(J|B)P(B)}{P(J)} \approx 0.0163$$

e) Calculate $P(A|J,M)$

$$\begin{aligned} P(A|J, M) &= \frac{P(A, J, M)}{P(J, M)} = \frac{P(A, J, M)}{P(A, J, M) + P(\neg A, J, M)} = \left(1 + \frac{P(\neg A, J, M)}{P(A, J, M)}\right)^{-1} \\ &= \left(1 + \frac{P(J|\neg A)P(M|\neg A)P(\neg A)}{P(J|A)P(M|A)P(A)}\right)^{-1} \approx 0.76 \end{aligned}$$